VALIDITY OF THE BOUNDARY CONDITIONS DEFINED FOR MEASUREMENTS OF THE HEAT TRANSFER COEFFICIENT ON THE BASIS OF THE THERMAL STRAIN RATE IN SPECIMENS

L. N. Linnik, V. S. Batalov, and K. I. Lobachev

UDC 536.242:536.2.083

The velocity of the dilatometric method of measuring the heat transfer coefficient for cylindrical surfaces in a transverse stream of liquid is confirmed theoretically.

Several fundamental premises in [1] do not seem adequately justified in physical terms. Specifically, one of the fundamental premises the practical validity of which is still much debated (especially in dilatometric studies of the heat transfer processes in liquids) is the absence of radial temperature gradients in standard (metallic) cylindrical specimens (an absence of axial temperature gradients is easily attained by extremely fast heating of specimens with electric current passing directly through the specimens and by making the measurements immediately after the electric heating current has been turned on, i.e., when the axial temperature gradients produced by contactive heat transfer through the end surfaces of a cylinder with a length-to-radius ratio l/R = 2-3 appear only across small segments adjoining these end surfaces).

Nevertheless, as this feasibility study of dilatometry based on certain theorems of thermoelasticity will show, all mathematical relations derived in [1] are fully applicable also when the necessary condition for heat transfer between the lateral surface of a cylinder and a surrounding liquid is the presence of radial temperature gradients inside a standard specimen.

One of the most important relations used in our analysis is the well-known theorem of thermoelasticity [2]:

$$U_l = 2\beta \frac{l}{R^2} \int_0^R T(r)\, rdr, \tag{1}$$

defining the absolute thermal deformation U_l of segment l of an infinitely long cylinder inside which there prevails an axially symmetrical temperature field T(r); the cylinder material is assumed isotropic, which makes coefficient β also an isotropic quantity.

It must be noted that even for infinitely long cylinders Eq. (1) is valid only if the temperature field in the cylinder is maintained axially symmetrical and not a function of the axial coordinate. As has been emphasized already, each of these requirements is satisfied for a sufficiently long specimen of the appropriate shape (a Π-shape, for example, where throughout the rather lengthy test duration there appear no axial gradients and dilatometric probes record only changes in the length of the top beam of the Π-specimen; in practice axial temperature gradients vanish immediately after the electric heating current has been turned off).

The rate at which one end surface of a cylinder is displaced (e.g., the $x = l$ end assumed unconstrained in space) with the other end $x = 0$ fixed is easily determined by differentiating Eq. (1) with respect to time:

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 22, No. 4, pp. 645-647, April, 1972. Original article submitted July 19, 1971.

$$W = \frac{dU_l}{dt} = 2\beta \frac{l}{R^2} \int_0^R \frac{\partial T}{\partial t} r dr. \tag{2}$$

The value of $\partial T/\partial t$ inside the cylinder can be found from the equation of heat conduction in cylindrical coordinates with an axially symmetrical temperature field [3]:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c_p} \cdot \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right). \tag{3}$$

Inserting this expression for $\partial T/\partial t$ from Eq. (3) into Eq. (2) and then integrating (2), we obtain

$$W = 2\beta \frac{\lambda l}{c_p R} \cdot \frac{\partial T}{\partial r}(R, t). \tag{4}$$

Considering that at the lateral cylinder surface there occurs heat transfer with the surrounding medium (whose temperature T_1 is assumed constant) according to the law

$$\lambda \frac{\partial T}{\partial r}(R, t) = \alpha \left(T(t) - T_1\right), \tag{5}$$

we will obtain for the linear expansion rate W at that instant of time

$$W = 2\alpha \frac{l}{c_p R} \beta \left(T(t) - T_1\right). \tag{6}$$

If W is measured immediately after the electric heating current has been turned on (at that instant there is contact established between the lateral cylinder surface and the liquid), then the temperature at all points inside the specimen will still be equal to the surface temperature T(t) and, therefore, the product of the two last factors on the right-hand side of Eq. (6) represents exactly the ultimate relative change of the cylinder dimensions from the time when the electric heating current has begun to flow till the end of the thermal expansion process with the entire specimen at temperature T_1 of the liquid.

In this way,

$$\beta \left(T(t) - T_1\right) = \frac{\Delta l}{l}. \tag{7}$$

In view of equality (7), it becomes worthwhile to rewrite expression (6) as

$$W = \frac{2\alpha \Delta l}{R c_p}. \tag{8}$$

From relation (8) follows directly the expression for calculating the heat transfer coefficient α:

$$\alpha = W \frac{R c_p}{2\Delta l}, \tag{9}$$

which is almost the same as the analogous formula in [1].

It is to be noted, in conclusion, that the relations derived in [1] are fully applicable also to calculations of heat transfer in liquids, and it is required only that the linear expansion rate W be measured immediately after the electric heating current has been turned on, i.e., at the instant when contact between the specimen and the surrounding medium is established.

NOTATION

c_p is the specific heat of the specimen material;
U_l is the absolute thermal linear expansion of the cylindrical specimen segment;
R is the radius of the cylindrical specimen;
T is the temperature, °K;
l is the specimen length;
α is the heat transfer coefficient;
λ is the thermal conductivity;
β is the thermal linear expansivity;
r is the radius, in cylindrical coordinates;
x is the axial distance, in cylindrical coordinates;
W is the rate of linear expansion.

LITERATURE CITED

1. V. S. Batalov, Inzh.-Fiz. Zh., 17, No. 1 (1969).
2. A. D. Kovalenko, Introduction to Thermoelasticity [in Russian], Naukova Dumka, Kiev (1965).
3. P. Schneider, Engineering Problems in Heat Conduction [Russian translation], IL, Moscow (1960).